

# Stability Analysis of Multiple-Robot Control Systems

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## Abstract

In a space telerobotic service scenario, cooperative motion and force control of multiple robot arms are of fundamental importance. In this paper, we propose three paradigms to study this problem. They are distinguished by the set of variables used for control design; the three possibilities are: joint torques, arm tip force vectors and the acceleration of a set of generalized coordinates. Control issues related to each case are discussed. The latter two choices require complete model information, which presents practical modeling, computational and robustness problems. We therefore focus on the joint torque control case to develop relatively model-independent motion and internal force control laws. The rigid body assumption allows the motion and force control problems to be independently addressed. By using an energy motivated Lyapunov function, we show that a simple proportional derivative plus gravity compensation type of motion control law is always stabilizing. The asymptotic convergence of the tracking error to zero requires the use of a generalized coordinate with the contact constraints taken into account. If a non-generalized coordinate is used, only convergence to a steady state manifold can be concluded. For the force control, both feedforward and feedback schemes are analyzed. The feedback control, if proper care has been taken, exhibits better robustness and transient performance.

## 1. Introduction

Spacecraft servicing by using autonomous telerobots has been under serious consideration for future deployment, such as the flight telerobotic servicer concept currently under study in NASA. In a typical telerobotic service scenario, a number of challenging control problems arise, including the control of open kinematic chains (arms moving into ready positions for servicing) and closed kinematic chains (arms handling a satellite, manipulating parts etc.), and attitude control (attitude of the platform that supports the arms, and attitude of the satellite). In this paper, we will address the issues related to the cooperative control of multiple rigid robot arms holding a commonly held object that is possibly in contact with a rigid surface.

A multiple-arm system can be viewed in different ways depending on the variables regarded as the control input in the controller design. Three levels of control paradigms can be constructed. On the first level, the joint torques are viewed as the control input. We call this perspective the full dynamics approach. On the second level, the tip forces are regarded as the control input and the joint torques are selected in a feedforward manner (which still requires real time joint angle measurements but has no error correction function) to realize the prescribed tip forces. We call this perspective the arms-as-actuator approach. On the third level, an unconstrained generalized acceleration (there are an infinite number of generalized coordinate representations for the constrained dynamics) is set equal to a pseudo-control input and the joint torques again generate the prescribed control action via a feedforward compensation. We call this perspective the feedback linearization approach.

By the nature of their structures, the last two approaches require the full dynamical model information to implement the feedforward compensation. However, the control law design is much simplified as the nonlinear dynamics of the arms are compensated. Since computational and robustness issues related to the multiple-arm control problem remain to be fully explored, we will focus on the full dynamics approach in this paper.

Due to the rigidity assumption on the held object, the grasp and the arms themselves, it is possible to decompose the tip contact force (of all arms collectively) into two orthogonal components, one that effects

motion of the held object and the other that produces a desired internal squeeze force. As a consequence, the motion and force control problems become decoupled (in one direction) in the following sense:

*Force control does not affect object motion; object motion does affect the internal force (due to the inertial, d'Alembert force).*

This motivates the following control design philosophy:

*Design a stable motion control law without the consideration of force control. Then design a stable force control law by treating the inertial force as a perturbation.*

Based on this philosophy, motion control laws for set point operation, with and without transient shaping, are developed by using a class of Lyapunov functions motivated by the total energy of the system. This class of control laws has an appealing simple structure of proportional and derivative feedback with gravity compensation. In the set point control of the internal force, both feedforward or feedback (if force/torque sensor information is available) strategies are analyzed. The feedback scheme has better robustness properties than the feedforward one, and, because of the motion/force decoupling property mentioned before, it can achieve tight transient control with high feedback gains.

For motion control, several choices of feedback variables are possible, with different implications in terms of performance, stability, relation to the control objective, amount of model information needed in implementation, etc. Here, we consider the joint variable, the tip variable and an unconstrained generalized coordinate. In all three cases, the proportional-derivative-gravity control law drives the system to a steady state configuration. However, only in the case of generalized coordinate does the steady state configuration correspond to the desired one. In the first two cases, the configuration lies in a manifold on which the tip forces produced by the arm position errors balance with one another. We call this manifold the jam manifold. Some preliminary results of its properties are discussed.

## 2. Model for Multiple-Arm Systems

All of the stability results discussed in this paper are based on the assumption that the arms and the held object are rigid and the grasp between the arms and the object is also rigid. Other models of multiple-arm systems sometimes insert a spring in the last link of each arm to simulate the effect of force/torque sensors. We feel that because the internal spring in the force/torque sensor is sufficiently rigid (implying small displacement) and the anticipated force transient is sufficiently benign (due to our force controller), our infinite rigidity assumption is a reasonable approximation. With the additional assumption that the object does not deform, we can decompose the tip force vector into two orthogonal components: one that contributes to motion of the system and one that builds up internal force. The component that effects motion is said to be in the "move" subspace and the component that builds up internal force is said to be in the "squeeze" subspace. This decomposition is appealing for several reasons. It agrees with human experience that squeeze forces can be applied without effecting any apparent motion. The analysis is simpler since it is free from the added complication of a spring in every arm, the effect of which does not appear to be very significant physically (if the spring is due to the force/torque sensor only and the internal force is controlled). There is also the possible application to task separation in combined autonomous/teleoperated types of operation (e.g. autonomous positioning and teleoperator force control). An important consequence of the rigidity assumption is that the motion and force control problems can be decoupled. The squeeze force control does not affect the motion of the held object, but the motion can affect the squeeze force. This motivates the following approach to control design: Design motion control first independent of the force control, then design force control by treating motion induced squeeze force (projection of the d'Alembert force in the squeeze subspace) as an external perturbation (which is unaffected by the squeeze control effort). The rigidity assumption also prevents direct proportional force feedback, for the algebraic loop results in an ill-posed dynamical system, destabilized by arbitrarily small delay in the force feedback channel; a filter with memory must be used instead. This issue will be addressed later in the section on force control.

With the assumptions stated above, the equation of motion are [1]:

$$M\ddot{q} = \tau - C\dot{q} - k - J^T f \quad (2.1a)$$

$$f_c = A^T f \quad (2.1b)$$

$$M_c \alpha_c = f_c + b_c + k_c \quad (2.1c)$$

$$\alpha = A\alpha_c + a = \dot{J}\dot{q} + J\ddot{q} \quad (2.1d)$$

$$v = Av_c = J\dot{q}$$

The symbols are defined as follows (an arm-related vector is composed of stacked single-arm vectors; an arm-related matrix is composed of block diagonalized single-arm matrices):  $q$  = joint variable,  $M$  = inertia matrix,  $C$  = arm coriolis and centrifugal force,  $k$  = arm gravity load,  $\tau$  = joint torque,  $f$  = tip force,  $J$  = arm Jacobian,  $A$  = object center of mass (CM) to arm tip Jacobian,  $f_c$  = force at object CM,  $b_c$  = object coriolis and centrifugal force,  $k_c$  = object gravity load,  $\alpha$  = tip acceleration,  $\alpha_c$  = object CM acceleration,  $v$  = tip velocity,  $v_c$  = object CM velocity,  $a$  = bias arm tip acceleration.

These equations can be combined to solve for the contact force,  $f$ , as

$$f = (AM_c^{-1}A^T + JM^{-1}J^T)^{-1}(\dot{J}\dot{q} - a - AM_c^{-1}(b_c + k_c) + JM^{-1}(\tau - C\dot{q} - k)) \quad (2.2)$$

$f$  is uniquely solvable if and only if  $[J \ A]$  has full rank. It was shown in [Cor.3.3 in 2] that if the manipulator system is kinematically parameterized by the Denavit-Hartenberg parameters and base positions of the arms, then  $[J \ A]$  having full rank at every kinematically feasible configuration is a generic property. Hence, we will assume that  $f$  is uniquely solvable. For the ease of presentation, we restrict our attention to the non-redundant arms case, though the redundant arms case can also be considered in the same framework.

The matrix  $A^T$  in (2.1b), which transforms the tip force to the force at object CM, is "fat", and therefore possesses a nontrivial null space. Since  $f_c = A^T f$ ,  $f$  in this null space means that it does not contribute to the motion of the held object but only to the buildup of internal forces. Hence, we define the squeeze subspace to be  $\mathbf{X}_s = \text{Ker}(A^T)$  (the kernel or the null space of  $A^T$ ). The orthogonal complement of  $\mathbf{X}_s$  is defined as the move subspace, which is  $\mathbf{X}_m = \text{Im}(A)$  (the image or the range space of  $A$ ) since  $\mathbf{R}^{m \times n} = \mathbf{X}_m \oplus \mathbf{X}_s$ . For a given tip force,  $f$ , there exists a unique orthogonal decomposition

$$f = f_m + f_s$$

where  $f_m \in \mathbf{X}_m$  and  $f_s \in \mathbf{X}_s$ . Only  $f_m$  contributes to the motion of the held object.  $A^T$  can be written as

$$A^T = [A_1^T, \dots, A_m^T]$$

where  $A_i^T$  transforms the tip force of arm  $i$  to the force at the object CM, and it is given by

$$A_i^T = \begin{bmatrix} I & \tilde{r}_{iC} \\ 0 & I \end{bmatrix}$$

where  $\tilde{r}_{iC}$  is the vector from the tip of arm  $i$  to the object CM and  $\tilde{\cdot}$  denotes cross product in a coordinate representation. Clearly,  $A_i^T$  is non-singular, and, hence,  $A^T$  is of full row rank. This implies that  $\dim(\mathbf{X}_s) = (m-1) \cdot n$  and  $\dim(\mathbf{X}_m) = n$ .

In (2.1 b-d), the tip forces of the arms can be regarded as actuator outputs. This leads to the arms-as-actuator approach. Eq. (2.2) can then be considered as a nonlinear compensator which computes the joint torques needed for the desired actuation signal. This viewpoint has also been adopted in [3].

For a selected set of unconstrained generalized coordinates, the multiple-arm dynamical equation can be partitioned in a different way so that the generalized acceleration is equal to a desired value. A nonlinear

feedforward filter then computes the joint torques required for the desired behavior. Denote the generalized coordinate and its kinematic relation to the joint angles by

$$\beta = h(q) \quad . \quad (2.3)$$

Then by differentiating twice with respect to  $t$  and denoting  $J_\beta(q) \triangleq \nabla_q h(q)$ , we have

$$\ddot{\beta} = u \quad (2.4a)$$

$$u = \dot{J}_\beta \dot{q} + J_\beta M^{-1}(\tau - C\dot{q} - k - J^T f) \quad , \quad (2.4b)$$

where  $f$  is given by  $\tau$ ,  $q$  and  $\dot{q}$  as in (2.2). We call the perspective of regarding  $u$  as the effective control input the feedback linearization approach. Note that (2.4) is valid irrespective of the redundancy of the arms.

### 3. Control Issues Related to the Feedback Linearization and the Arms-As-Actuator Approaches

In the arms-as-actuator paradigm, the dynamics involving the held object seen at the center of mass are composed of two parts: a force balance equation (Newton's equation) and a torque balance equation (Euler's equation). The force equation is linear and can be controlled easily. The torque equation involves control on the rotation group,  $SO(3)$ . Various possible control laws for the latter problem have been analyzed in [4, wkattcdc]. In particular, a control law involving the feedback of the unit quaternion of the attitude error can be used for globally asymptotically stable closed loop operation.

In the feedback linearization paradigm, the control law can be easily constructed since the feedback linearized system is in double integrator form. However,  $J_\beta$  in the control law introduces additional singularities which are a mathematical constraint rather than a physical limitation.

The non-uniqueness of  $f$ , and therefore  $\tau$ , for controlling the object motion can be used to control the closed chain internal forces. This can be posed as an optimization problem, giving rise to the load-balancing problem as discussed in [6].

Full dynamical model information is needed in the nonlinear feedforward compensation for both the feedback linearization and arms-as-actuator schemes. The computational and robustness issues due to the complex nonlinear, model dependent compensation need to be addressed for a successful implementation. At the present, work in this direction is lacking in the context of multiple-arm systems. For this reason, the rest of the paper will focus on the full dynamics approach and develop relatively model-independent control laws directly for the joint torques.

### 4. Control Issues Related to the Full Dynamics Approach

In this section, the full dynamical model is analyzed to develop motion and force control strategies. A consequence of the move/squeeze decomposition is that any term in  $\tau$  of the form  $J^T F_s$ , with  $F_s$  in the squeeze subspace, does not affect motion of the system. However, the motion of the system does affect the actual squeeze force, due to the squeeze component of the d'Alembert (inertial) forces. This motivates the following decomposition of the control torque:

$$\tau = \tau_m + \tau_s + \tau_g \quad , \quad (4.1)$$

where  $\tau_m$  is responsible for the motion control,  $\tau_s$  is responsible for the squeeze force control and  $\tau_g$  compensates for the gravity load due to the arms and the held object. In Section 4.1, various possible motion feedback control laws, based on the variables used for feedback, are discussed. In Section 4.2, different force control laws are discussed.

For motion control, we propose a class of relatively model independent control laws that have a simple Proportional Derivative (PD) plus gravity compensation type of structure. For the internal force control,

asymptotically stable, model independent, set point controllers based on feedforward or feedback implementation are constructed. The feedback controller is shown to possess superior robustness property and transient performance.

#### 4.1 Motion Control

It has been shown [7] that PD feedback plus gravity compensation is a globally asymptotically stable set point control law for a single arm with an unconstrained tip. The feedback variable can be either the joint variable (angle or displacement) or the tip variable (tip position and orientation, the latter being suitably parameterized). For tip variable feedback, Jacobian non-singularity for all time is assumed. The structure of this class of control law is very appealing since it is relatively model independent (only arm gravity compensation is needed) and has an energy dissipation interpretation. Therefore, it is reasonable to investigate its generalization to the multiple-arm case.

For a multiple-arm system, there are three possible types of feedback variables: 1. joint position (of all arms), 2. tip position (of all arms), 3. a generalized coordinate. The first two over-specify the configuration of the system (due to the kinematic constraint imposed by the rigid grasp of a common object) and hence are not generalized coordinates. The ramification of using them for feedback will be discussed later. Possible choices for generalized coordinates include position and orientation of the mass center of the held object, a subset of the tip position, joint position and/or tip forces. For tip position feedback and generalized coordinate feedback involving orientation, a parameterization of orientation needs to be chosen. We will assume that a minimal representation is used, though other related works [4,5] have indicated that the unit quaternion (Euler parameters) may be a better choice since there is no problem with the singularity of representation.

We will address point-to-point control only. Generalization to the general tracking problem is currently under investigation. If the transient performance is not expressly considered, then, as shown in [8,9], a straightforward generalization of the single arm energy Lyapunov function approach shows that a steady state is reached when PD plus gravity type of control law is used for all three types of feedback variables. In the cases of joint level and tip level feedback, the steady state error converges to a manifold, even if the set point represents a kinematically feasible configuration. Only in the generalized coordinate feedback case is the steady state error zero.

The fixed-set-point control laws in [8,9] are useful in demonstrating the application of a general class of Lyapunov functions and pointing out some interesting issues unique to the multiple-arm control problem (jam manifold, squeeze force control, etc.). However, the fixed set point control paradigm is fundamentally flawed because the closed loop trajectory transient is not controlled. For initial condition far away from the desired set point, the transient is typically so wild, these control laws are virtually unusable. The problem is most severe in tip and generalized coordinate feedback, where arms may cross Jacobian singularities, flip poses (due to multiple solutions to the inverse kinematics problem), collide with themselves, violate joint stops, etc. This motivated us to extend our framework to include trajectory shaping in the set point operation.

A possible method to shape transient performance is to replace the position error and the velocity in the fixed-set-point control laws by the difference between the actual trajectory and a desired trajectory to the goal set point which is chosen to have good transient behavior. Since the desired trajectory converges to a set point, we shall call it the moving set point. Intuitively, we expect better transient response with the moving set point controller since the applied torque increases gradually rather than abruptly. In this section, we will show that provided the desired velocity and acceleration satisfy some mild conditions, the moving set point controller results in the same closed loop stability property as in the fixed set point case.

Assume the desired trajectory has the following properties:

$$\left\{ \begin{aligned} & q_{des}(t) \rightarrow \text{constant}, \quad \dot{q}_{des}(t) \rightarrow 0, \quad \ddot{q}_{des}(t) \rightarrow 0, \\ & \dot{q}_{des} \in L_2, \quad \ddot{q}_{des} \in L_2 \cap L_\infty, \quad \text{both } \ddot{q}_{des}, \quad \ddot{q}_{des} \text{ are continuous.} \end{aligned} \right\}$$

These are mild restrictions on the desired trajectories. If the desired trajectory possesses these properties, then we say the desired trajectory belongs to the class  $\mathcal{S}$ . We modify the Lyapunov function used for fixed-set-point control by replacing velocities by velocity tracking errors:

$$V = \frac{1}{2} \Delta v_c^T M_c \Delta v_c + \frac{1}{2} \Delta \dot{q}^T M \Delta \dot{q} + U^* \quad (4.2)$$

where

$$\Delta v_c = v_c - v_{c_{des}}, \quad v_{c_{des}} = A^+ v_{des}, \quad v_{des} = J(q) \dot{q}_{des}, \quad \Delta \dot{q} = \dot{q} - \dot{q}_{des},$$

and  $U^*$  is given below

Type of Feedback	$U^*(q)$
Joint Level	$\frac{1}{2} \Delta q^T K_p \Delta q$
Tip Level	$\frac{1}{2} \Delta x^T K_p \Delta x$
Generalized Coordinate	$\frac{1}{2} \Delta \beta^T K_p \Delta \beta$

Table 1. Quadratic Potential Energy Candidates

The generalized velocity,  $\dot{\beta}$  is related to the tip velocity via  $v = B\dot{\beta}$ . The superscript  $+$  denotes Moore-Penrose generalized inverse. Suppose the following proportional-derivative-gravity control law is used for the motion control (cf. (4.1)):

$$\tau_m + \tau_g = -\tau_p - \tau_v + k(q) - J^T F_c, \quad (4.3)$$

where  $k$  is the arm gravity load, and  $F_c$  is gravity compensation for the held object, chosen to satisfy  $A^T F_c = k_c$ . The proportional and velocity feedback terms are given in the table below, depending on the variables used for feedback:

Type of Feedback	$\tau_p$	$\tau_v$
Joint Level	$K_p \Delta q(t)$	$K_v \Delta \dot{q}(t)$
Tip Level	$J^T K_p \Delta x(t)$	$J^T K_v \Delta v(t) + D \Delta \dot{q}(t) \quad D > 0$
Generalized	$J^T F_p$	$J^T F_D + D \Delta \dot{q}(t) \quad D > 0$
Coordinate	$B^T F_D = K_p \Delta \beta(t)$ $B^T F_D = K_p \Delta \dot{\beta}(t)$	

Table 2 PD Feedback in Moving Set Point Motion Control Laws

Then the derivative of  $V$  along the solution can be bounded by

$$\dot{V} = -\lambda \|\Delta \dot{q}\|^2 + \eta_1(t) \|\Delta \dot{q}\|^2 + \eta_2(t) \|\Delta \dot{q}\| \quad (4.4)$$

where  $\eta_1(t) \rightarrow 0$  as  $t \rightarrow \infty$  and  $\eta_2 \in L_2[0, \infty)$  due to the assumed properties of the desired trajectory. By integrating both sides from  $t_o$  to  $t$ , for  $t_o$  sufficiently large, there exists  $\lambda_1 > 0$  such that

$$\begin{aligned} \lambda_1 \|\Delta \dot{q}\|_{L_2([t_o, t])}^2 &\leq V(t_o) - V(t) + \int_{t_o}^t \eta_2(\tau) \|\Delta \dot{q}(\tau)\| d\tau \\ &\leq V(t_o) + \|\eta_2\|_{L_2([t_o, t])} \|\Delta \dot{q}\|_{L_2([t_o, t])} \end{aligned} \quad (4.5)$$

Now, by completing the squares involving  $\|\Delta\dot{q}\|_{L_2([t_0, t])}$ , it follows that  $\Delta\dot{q} \in L_2([t_0, \infty))$ . From (4.5),  $V(t)$  is uniformly bounded for all  $t$ , which implies  $\Delta q$ ,  $\Delta\dot{q}$ ,  $\Delta\ddot{q}$  and  $\Delta\ddot{q}$  are uniformly bounded. By Barbalat's Lemma [10],  $\Delta\dot{q} \rightarrow 0$  as  $t \rightarrow \infty$ , which, by Lemma 1 of [11], implies  $\Delta\ddot{q} \rightarrow 0$ , also. Now, from the arm dynamical equation,  $\tau_p + J^T F_c + J^T f \rightarrow 0$  which yields the same convergence result as in the fixed set point case.

The stability properties of the moving set point controllers can now be summarized below:

**Result 4.1.** If the desired trajectory belongs to class  $\mathcal{S}$ , then the multiple-arm system with control laws (4.3) and Table 2 has the following stability property:

<u>Type of Feedback</u>	<u>Type of Stability</u>
Joint Level	$\dot{q} \rightarrow 0$ , $\Delta q$ converges to the manifold $K_p \Delta q + J^T (F_c + f) = 0$ (4.6)
Tip Level	$v \rightarrow 0$ , $\Delta x$ converges to the manifold $J^T (K_p \Delta x + F_c + f) = 0$ (4.7)
Generalized Coordinate	$\beta, \dot{\beta} \rightarrow 0$ (global asymptotic stability).

Table. 3 Stability Properties of Fixed Set Point Motion Controllers

Furthermore, if the initial tracking error is zero, the maximum trajectory tracking error is inversely proportional to the size of the PD gains. ■

If the desired trajectory starts with the same initial condition as the actual trajectory and has the desired transient behavior, e.g., small overshoot, no excessive acceleration or jerk, avoiding Jacobian singularities, joint stops, obstacles and the arms themselves, Result 4.1 shows that high enough feedback gains ensure that the actual trajectory will have similar properties, also. The maximum tracking error can be shown proportional to the  $L_2$ -norm of  $\eta_2$  in  $\dot{V}$  which is composed of the difference between the moving set point and its steady state, the desired velocity and acceleration. A trajectory planning problem can be posed to find a desired trajectory that satisfies the required transient response and minimizes the  $L_2$ -norm of  $\eta_2$ .

Control laws that incorporate the full model information can also be constructed within this approach (by using, for example, results in [12]). We can qualitatively state the advantage of this added complexity in the control algorithm. In the tracking control problem, even if the initial tracking error is zero, the PD control law will always incur a nonzero trajectory tracking error. This error can be made small if gains are allowed to be large; however, it may not always be practical, given the limited actuator size and the noise problem. With the model-dependent control laws, the tracking error will remain zero (at least theoretically; noise will cause small deviation from the desired trajectory). The same is true in the internal force control (see next section). If the full model information is assumed, precise force control at every moment in time is possible; while the model-independent control law reduces finite force error with high gains. The type of control law to use for a given application depends on the trade-off between the available a priori model information and the performance requirement, subject to actuator and sampling constraints.

## 4.2 Force Control

In this section, we consider the problem of choosing  $\tau_s$  in (4.1) to asymptotically drive the squeeze force to a desired set point. To ensure that arm motion is not affected by the squeeze force control, we choose

$$\tau_s = J^T F_s \quad (4.8)$$

where  $F_s$  is restricted to lie in the squeeze subspace. The effective control variable for force control is now  $F_s$ .

Without using the full model information, the squeeze force can only be controlled asymptotically. (If model information is available, an additional optimal load distribution problem can be posed; see [6].) We will show that either feedforward or feedback control structure may be used to drive the squeeze force to its set point asymptotically, depending on whether force sensors are available. For the full composite force vector,  $m$  force sensors need to be used. The feedback strategy, if properly applied, has better performance and robustness. If the gravity load in the squeeze subspace is not fully compensated, feedback control can still be used but the feedforward control will incur a squeeze force error equal to the squeeze component of the gravity error.

By projecting the composite tip force vector, given by (2.2), in the squeeze subspace, and applying (4.8), we have

$$f_s = F_s + \eta \quad (4.9)$$

where  $\eta$  represents the projection of the inertial force in the squeeze subspace. Recall that an important property of  $\eta$  due to the move/squeeze decomposition is that it is not affected by  $F_s$ . Hence, it can be treated as an external disturbance. We assume that one of the PD type of control strategies in Section 4.1 has been used for motion control. (In fact, any stable motion control law can be used without affecting the subsequent argument.) Then  $\eta(t) \rightarrow 0$  as  $t \rightarrow \infty$ . We are interested in studying the following aspects of the force control problem:

1. Stability. (Does  $f_s(t) \rightarrow f_{s_{des}}$  as  $t \rightarrow \infty$  in (4.9)?)
2. Transient performance. (What is the maximum force error, i.e.,  $\max_{t \geq 0} f_s(t) - f_{s_{des}}$ ?)
3. Convergence rate. (How fast does  $f_s \rightarrow f_{s_{des}}$ ?)
4. Noise reduction. (If  $\eta(t) \rightarrow \eta_\infty \neq 0$ , representing a persistent noise, what is the steady state force error?)

To attain asymptotic stability, a feedforward control will clearly suffice

$$F_s = f_{s_{des}} \quad (4.10)$$

However, the transient performance and convergence rate are determined entirely by  $\eta$  (which are in turn determined by the quality of the motion control law). There is also no noise reduction in this scheme.

If the arm tip forces are measured, then clearly a feedback strategy is preferable, due to the hope for added insensitivity to noise and improved transient performance. However, the infinite rigidity assumption stated in section 2.1 necessitates extra care in the control design. We will show that the lack of dynamics in (4.9) means that infinite bandwidth feedback from  $f_s$  to  $F_s$  would violate the strict causality of the loop. This has some unintended consequences. For example, the control law

$$F_s = f_{s_{des}} + \beta(f_s - f_{s_{des}}) \quad (4.11)$$

implies  $f_s \rightarrow f_{s_{des}}$  for  $\beta \neq 1$ . Furthermore, transient performance, convergence rate and steady state error due to noise can all be much improved over the feedforward case, if  $\beta$  is large. However, an arbitrarily small time delay in the feedback channel (which is always present in a physical implementation) leads to instability if  $|\beta| > 1$ . If  $|\beta| < 1$ , then the response of the resulting linear discrete time system consists of two terms: the homogeneous solution and the particular solution due to  $\eta$ . For fast convergence of the homogeneous solution to zero,  $\beta$  needs to be close to zero, but then the response is similar to that of the feedforward control and the desirable properties due to the force feedback is lost.

Recognizing that the problem is caused by the algebraic loop due to the proportional force feedback, we suggest pre-processing the measured force by a strictly causal filter (if the filter is linear, then strictly proper). The feedback control law then takes on the following form:

$$F_s = f_{s_{des}} + C(f_s - f_{s_{des}}) \quad (4.12)$$



where  $C$  is a strictly proper linear filter such that  $(I - C)$  has zeros only in the open left half plane. Clearly,  $f_s(t) \rightarrow f_{s,ss}$  as  $t \rightarrow \infty$ . To see the transient behavior, we write

$$\Delta f_s \triangleq f_s - f_{s,ss} = \mathcal{L} * \eta \quad (4.13)$$

where  $*$  denotes the convolution operator and  $\mathcal{L}$  the convolution kernel associated with  $(I - C)^{-1}$ . Since  $\mathcal{L}$  can contain arbitrarily fast dynamics (if the desired dynamics of  $\mathcal{L}$  is  $\frac{a(s)}{b(s)}$  in the Laplace domain, then  $C(s) = \frac{a(s)-b(s)}{a(s)}$  is the Laplace transform of the corresponding filter  $C$ ), the  $L_1$  norm of  $\mathcal{L}$  can be made arbitrarily small. By the following error estimate [Appendix C,13],

$$\|\Delta f_s\|_{L_\infty} = \|\mathcal{L}\|_{L_1} \|\eta\|_{L_\infty}$$

it follows that the transient performance and convergence rate can both be improved. To see the effect of a persistent noise, we apply the initial value theorem:

$$\lim_{s \rightarrow 0} s \Delta f_s(s) = \lim_{t \rightarrow \infty} \Delta f_s(t) \quad (4.14)$$

We conclude that if  $C(s)$  has a pole at the origin, then there is no steady state error even if  $\eta$  does not converge to zero. Hence, all the control objectives are satisfied with the control law (4.12), provided that  $(I - C)^{-1}$  is a stable filter and  $C$  has a pole at the origin. If the spectrum of  $\eta$  is known (say, for repeated tasks),  $C$  can be chosen to selectively notch out the dominant dynamics in  $\eta$ . What about robustness with respect to small time delays? To address this problem, we use a first order approximation of  $f_s(t - \Delta t)$ , for  $\Delta t$  small, i.e.,

$$f_s(t - \Delta t) \approx f_s(t) - \Delta t \dot{f}_s(t) \quad .$$

The closed loop system in the Laplace domain now becomes

$$\Delta f_s(s) = (I - C(s) - \Delta t C(s)s)^{-1} \eta(s) \quad (4.15)$$

Since  $C(s)$  is strictly proper and  $(I - C)^{-1}$  is stable, for sufficiently small  $\Delta t$ , the perturbed system (4.15) remains stable. In the case of direct proportional feedback,  $C$  is not strictly proper and indeed the corresponding closed loop system becomes unstable for arbitrarily small  $\Delta t$ .

A particularly simple choice of  $C$  is just an integrator, i.e., in the Laplace domain,

$$C = -\frac{\beta}{s} \quad (4.16)$$

This control law has all the desirable features discussed above. If the integral feedback gain  $\beta$  is chosen sufficiently large, and  $\dot{\eta}(t)$  is uniformly bounded in  $t$ , then by explicitly solving the closed loop dynamical equation, it can be shown that the transient effect of  $\eta$  on  $f_s - f_{s,ss}$  can be made arbitrarily small.

The discussion in this section can be summarized in the result below:

**Result 4.2.** For the multiple-arm control system under consideration, if the arm configuration converges to a steady state (i.e., velocity converges to zero), then either the feedforward controller (4.11) or the feedback controller (4.12), with  $C$  a strictly proper linear filter and  $(I - C)$  containing zeros only in the open left half plane, drives  $f_s \rightarrow f_{s,ss}$ .

If in (4.12),  $C$  has a pole at the origin, then replacing  $\tau_g$  in (4.1) by its projection in the move subspace does not affect the asymptotic convergence of  $f_s - f_{s,ss}$  to zero, and, in general,  $f_s \rightarrow f_{s,ss}$  even if  $\eta \rightarrow \eta_\infty \neq 0$ .

If  $C$  is chosen to be an integrator as in (4.16) and  $\eta$  in (4.15) has a uniformly bounded time then  $f_s(t) - f_{s,ss}$  tends to zero uniformly for  $t$  in bounded intervals as  $\beta \rightarrow \infty$ . ■

## 5. Conclusion

This paper has considered several possible control structures for multiple-arm systems by regarding either joint torques, tip force, or a generalized acceleration as the control input. We have mainly focused on the first case since a class of relatively model-independent control laws can be generated for both motion and internal force control. The recently developed move/squeeze orthogonal subspace decomposition coupled with the energy Lyapunov function formulation provides the basic analytical framework. Future research includes the tuning of PD gains to improve tracking performance and generalizations to the multiple degrees-of-freedom contact case.

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